Manifestly Covariant Relativity [1]

Kenneth Dalton

e-mail: kxdalton@yahoo.com

Abstract

According to Einstein's principle of general covariance, all laws of nature are to be expressed by manifestly covariant equations. In recent work, the covariant law of energy-momentum conservation has been established. Here, we show that this law gives rise to a fully covariant theory of gravitation and that Einstein's field equations yield total energy-momentum conservation.

[1]This paper appeared in the *Hadronic Journal* **17**, 139 (1994). It pre-dates the four related articles which are listed on page 5.

> "The general laws of nature are to be expressed by equations which hold good for all systems of coordinates, that is, are covariant with respect to any substitutions whatever (generally covariant)." [1]

The great beauty of Einstein's theory of general relativity has been justly celebrated over the decades. The mathematical beauty of the theory consists in the property of manifest covariance. The gravitational field equations are covariant, as are the accompanying equations of motion and Maxwell's field equations. The stress-energy-momentum of matter and electromagnetism form covariant expressions, as well. Finally, the covariant Bianchi identities provide closure to the overall geometric structure. Nevertheless, there comes a point at which the beauty and simplicity of the theory abruptly end. In his treatment of energy-momentum, Einstein expressed the differential law of conservation in terms of the ordinary divergence [2]

$$\frac{\partial \sqrt{-g}\, T^{\mu\nu}}{\partial x^{\nu}} = 0 \tag{1}$$

This gave rise to the gravitational stress-energy-momentum pseudotensors [1–6]. These energy pseudotensors are decidedly lacking in simplicity. They are not covariant expressions, therefore the components can take on arbitrary values, including zero [3,4]. The conclusion has been reached that the pseudotensors can have no physical meaning [3], and that no definite statement can be made regarding the density of gravitational energy in any given region of space-time [5]. It is said that gravitational energy is 'nonlocalizable', that it is not observable [6], and that it is meaningless to talk of whether or not there is gravitational energy at a given place [2]. All of this stands in marked contrast to the stress-energy-momentum of matter and electromagnetism, which is both perfectly well-defined and physically observable.

Recently, it has been shown that equation (1) does not properly account for energy-momentum. The correct expression of conservation is given by the covariant divergence [7,8]

$$T^{\mu\nu}_{;\nu} = \frac{1}{\sqrt{-g}} \frac{\partial \sqrt{-g}\, T^{\mu\nu}}{\partial x^{\nu}} + \Gamma^{\mu}_{\nu\lambda} T^{\nu\lambda} = 0 \tag{2}$$

This equation arises from the vector divergence formula

$$\oint \mathbf{e}_{\mu} \sqrt{-g}\, T^{\mu\nu}\, d^3V_{\nu} = \int \mathbf{e}_{\mu} \sqrt{-g}\, T^{\mu\nu}_{;\nu}\, d^4x \tag{3}$$

and continuity equation

$$T^{\mu\nu}_{;\nu} = 0 \tag{4}$$

It has also been derived by variation of the action

$$S = \int L\sqrt{-g}\, d^4x \tag{5}$$

under uniform displacement in space-time [7]. This new law of conservation is the key to forming a fully covariant theory of gravitation. We begin with Einstein's gravitational field equations

$$R^{\mu\nu} - \frac{1}{2}g^{\mu\nu}R = -\frac{8\pi G}{c^4}T^{\mu\nu} \tag{6}$$

$T^{\mu\nu}$ represents the stress-energy-momentum of electromagnetism and matter. The covariant divergence of the left-hand side is identically zero, therefore

$$T^{\mu\nu}_{;\nu} = 0 \tag{7}$$

This equation means that the energy-momentum of matter and electromagnetism is conserved. In other words, there is no exchange of energy-momentum with the gravitational field. The inevitable conclusion to be drawn from this is that the gravitational field, itself, has no dynamical content—no energy, momentum, or stress. The elimination of the energy pseudotensor removes the last vestige of non-covariance from Einstein's theory of gravitation. All physically meaningful expressions are now manifestly covariant.

The absence of gravitational field energy implies that gravitational forces do not exist. We can demonstrate force-free planetary motion by means of a direct appeal to the equations of motion. The equation of motion for charged matter is found by substituting the energy tensors of matter and electromagnetism into equation (7) and then making use of Maxwell's equations:

$$\rho c^2\left(u^\nu \frac{\partial u^\mu}{\partial x^\nu} + \Gamma^\mu_{\nu\lambda}u^\nu u^\lambda\right) + F^\mu{}_\alpha J^\alpha = 0 \tag{8}$$

The path of an uncharged particle satisfies the geodesic equation

$$\frac{du^\mu}{ds} + \Gamma^\mu_{\nu\lambda}u^\nu u^\lambda = 0 \tag{9}$$

Einstein has stated that the term $\Gamma^{\mu}_{\nu\lambda} u^{\nu} u^{\lambda}$ is to be interpreted as follows: "The gravitational field transfers energy and momentum to the matter, in that it exerts forces upon it and gives it energy" [9]. However, all connection coefficients $\Gamma^{\mu}_{\nu\lambda}$ can be transformed to zero in any given infinitesimal region (geodesic coordinates) [10]. Therefore, no real physical transfer of energy-momentum may be assigned to this term. (By way of contrast, the Lorentz force $F^{\mu}{}_{\alpha} J^{\alpha}$ is covariant and cannot be transformed to zero.) We conclude that a planet moves through the gravitational field in force-free geodesic motion. The field itself is inherently curved, and this is revealed by the curved planetary trajectory. Only a real physical force, such as the Lorentz force, could transfer energy-momentum to the planet and cause it to deviate from its geodesic path.

The components of the matter tensor $T^{\mu\nu} = c^2 \rho u^{\mu} u^{\nu}$ obviously change during geodesic motion. The question then arises as to whether the energy-momentum of the planet is, in fact, conserved. The physical density of stress-energy-momentum is not given merely by the above tensor components, but by the coordinate-independent expression

$$\mathbf{T} = \mathbf{e}_{\mu} \otimes \mathbf{e}_{\nu} \, T^{\mu\nu} \tag{10}$$

In order to investigate conservation, we form the divergence [11]

$$\boldsymbol{\nabla} \cdot \mathbf{T} = \boldsymbol{\nabla} \cdot (\mathbf{e}_{\mu} \otimes \mathbf{e}_{\nu} \, T^{\mu\nu}) = \mathbf{e}_{\mu} \, T^{\mu\nu}_{;\nu} \tag{11}$$

This is zero by virtue of the equation of motion. As a planet moves along its orbit, the basis vectors $\mathbf{e}_{\mu}$ change in magnitude and direction such that $\boldsymbol{\nabla} \cdot \mathbf{T} = 0$ and the energy-momentum of the planet is conserved. Similar considerations apply to time-dependent gravitational fields, in general, and to the emission and absorption of gravitational waves, in particular. Conservation of total energy-momentum is assured, if Einstein's gravitational field equations are satisfied.

References

1. A. Einstein, "The Foundation of the General Theory of Relativity" in *The Principle of Relativity*, (Dover, New York 1952).

2. L.D. Landau and E.M. Lifshitz, *The Classical Theory of Fields*, (Pergamon, 4th edition, 1975) section 96.

3. W. Pauli, *The Theory of Relativity*, (Dover, New York, 1981) section 61.

4. H. Weyl, *Space-Time-Matter*, (Dover, New York, 1952) section 33.

5. R. Adler, M. Bazin, and M. Schiffer, *Introduction to General Relativity*, (McGraw-Hill, New York, 1965) page 314.

6. C. Misner, K. Thorne, and J. Wheeler, *Gravitation*, (Freeman, New York, 1973) page 467.

7. K. Dalton, *Gen.Rel.Grav.*, **21**, 533 (1989).

8. J. Vargas and D. Torr, *Gen.Rel.Grav.*, **23**, 713 (1991).

9. A. Einstein, *The Meaning of Relativity*, (Princeton, 1953) page 83; A. Einstein (Ref. 1) page 151.

10. L.D. Landau and E.M. Lifshitz (Ref. 2) page 241.

11. C. Misner, K. Thorne, and J. Wheeler, (Ref. 6) page 261.

Related works by the same author (at www.arxiv.org):

- "Farewell to General Relativity" physics/9710001
- "Einstein's Violation of General Covariance" physics/9703023
- "Gravity, Geometry, and Equivalence" gr-qc/9601004
- "Einstein's Energy-Free Gravitational Field" gr-qc/9512008